namibia university
OF SCIENCE AND TECHNOLOGY
FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 08BSHM	LEVEL: 8
COURSE CODE: AOR802S	COURSE NAME: APPLIED OPERATIONS RESEARCH
SESSION: JANUARY 2023	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 240 (To be Converted to 100%)

SUPPLEMENTARY/2ND OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	PROF. S. A. REJU
MODERATOR:	PROF. O. D. MAKINDE

INSTRUCTIONS
1. Attempt ALL the questions.
2. All written work must be done in blue or black ink and sketches must be done in pencil.
3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (including this front page)

QUESTION 1 [84 MARKS]

(a) Consider a winning bid of $5.4 million to construct a new plant for a major manufacturer and the manufacturer needs the plant to go into operation within 40 weeks. Below is the list of the various project activities. The third column provides important additional information for coordinating the scheduling of the project crews.

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	A	4 weeks
C	Put up the rough wall	B	10 weeks
D	Put up the roof	C	6 weeks
E	Install the exterior plumbing	C	4 weeks
F	Install the interior plumbing	E	5 weeks
G	Put up the exterior siding	D	7 weeks
H	Do the exterior painting	E, G	9 weeks
I	Do the electrical work	C	7 weeks
J	Put up the wallboard	F, I	8 weeks
K	Install the flooring	J	4 weeks
L	Do the interior painting	J	5 weeks
M	Install the exterior fixtures	H	2 weeks
N	Install the interior fixtures	K, L	6 weeks

(i) Define Critical Path Method (CPM) and Project Evaluation and Review Technique (PERT).

(3.5 Marks)

(ii) Sketch the project network diagram for the above project. (16 Marks)

(iii) Distinguish between crashing a project and a project activity. Hence obtain the crash costs per week saved for each activity from the following investigative time-cost trade-off data. (37 Marks)

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Activity	Time Normal	Time Crash	Cost Normal	Cost Crash
A	2 weeks	1 week	$180,000	$ 280,000	I	7 weeks	5 weeks	$210,000	$ 270,000
B	4 weeks	2 weeks	$320,000	$ 420,000	J	8 weeks	6 weeks	$430,000	$ 490,000
C	10 weeks	7 weeks	$620,000	$ 860,000	K	4 weeks	3 weeks	$160,000	$ 200,000
D	6 weeks	4 weeks	$260,000	$ 340,000	L	5 weeks	3 weeks	$250,000	$ 350,000
E	4 weeks	3 weeks	$410,000	$ 570,000	M	2 weeks	1 week	$100,000	$ 200,000
F	5 weeks	3 weeks	$180,000	$ 260,000	N	6 weeks	3 weeks	$330,000	$ 510,000
G	7 weeks	4 weeks	$900,000	$1,020,000					

(iv) Discuss your observations. (4 Marks)

(b) Consider a flow network with a directed graph with three vertices and three arcs, described as follows: The first arc from vertex(1) to vertex(2) has capacity 3 and the cost 1; the second arc from vertex(1) to vertex(3) has capacity 5 and the cost 4, and the third arc from vertex(2) to vertex(3) has capacity 4 and the cost 2.

(i) Sketch the flow network. (12 Marks)

(ii) State the matrices for the arc capacities, the arc costs and the demand function for the vertices. (11.5 Marks)

QUESTION 2 [50 MARKS]

(a) Define the strategic form of a 2-Person zero-sum game, explicitly explaining its meaning.

(4 Marks)

(b) Discuss the Reduction by Dominance procedure and hence simplify by using reduction by dominance the game defined by the following payoff matrix, showing progressively the reduced pay-off matrix: (18 Marks)

Player B

$$
\text{Player A} \quad
\begin{array}{c}
A \\ B \\ C \\ D
\end{array}
\begin{bmatrix}
1 & -1 & -5 \\
4 & -4 & 2 \\
3 & -3 & -10 \\
5 & -5 & -4
\end{bmatrix}
$$

(c) Distinguish between pure and mixed strategies. (6 Marks)

(d) Susan has a 250-gallon capacity home heating oil tank, presently empty, meant to store oil against the next winter. Consider the following winter heating oil quantity needed and the oil prices during probable four levels of winter severity:

WINTER SEVERITY	OIL STORAGE NEEDED	OIL PRICES PER GALLON
Mild Winter (MW)	110 Gallons	N$1.00
Average Winter (AW)	180 Gallons	N$1.85
Severe Winter (SW)	230 Gallons	N$2.00
Prolonged Winter (PW)	250 Gallons	N$3.00

Formulate a game model and employ the Minimax criterion technique to determine the gallons of oil Susan should stockpile at the current price of N$1 per gallon to avoid oil wastage and to maximise her saving. (22 Marks)

QUESTION 3 [53 MARKS]

(a) Consider a competition between two companies, Coca-Cola and Pepsi, and assume the former is thinking of cutting the price of its iconic soda. If it does so, Pepsi may have no choice but to follow suit for its cola to retain its market share. This may result in a significant drop in profits for both companies. Let's assume that the incremental profits that accrue to Coca-Cola and Pepsi are as follows: If both keep prices high, profits for each company increase by $500 million (because of normal growth in demand). If one drops prices (i.e. defects) but the other does not (i.e. cooperates), profits increase by $750 million for the former because of greater market share and are unchanged for the latter. If both companies reduce prices, the increase in soft drink consumption offsets the lower price, and profits for each company increase by $250 million.

   (i) Considering the above as an example of applications of Prisoner's dilemma problem, construct the payoff matrix for each company and for the game model, taking Coca-Cola as the row player. (16 Marks)

   (ii) What should each company do? (7 Marks)

(b) Consider a construction firm that is deciding to specialise in building High School blocks or Elementary School blocks or a combination of both. The construction company must submit a bid proposal, which costs money to prepare, and there are no guarantees that it will be awarded the contract. If the company bids on the high school, it has a 35% chance of getting the contract, and it expects to make $162,000 net profit. However, if the company does not get the contract, it loses $11,500. If the company bids on the elementary school, there is a 25% chance of getting the contract, and it would net $140,000 in profit. However, if the company does not get the contract, it will lose $5,750.

(ii) What should the construction company do? (14 Marks)

(iii) How sensitive to the estimate of the probability of the award of a contract is the decision (i):
- in either to build the High School or the Elementary School blocks? (6.5 Marks)
- to the net profit for each case, if awarded the contract? (9.5 Marks)

QUESTION 4 [53 MARKS]

(a) Provide a comprehensive definition of a Decision tree and hence diagrammatically show its basic characteristic components. (14 Marks)

(b) Using the problem in Question 2(c) above, provide the Fold-Back method tree for its solution. (14 Marks)

(c)

(i) What is the Kendall's classification of Queuing Systems? (5 Marks)

Discuss specifically the M/M/1 queuing system and the process $N(t)$ describing its state at time t as a birth-death process. Provide its state independent parameter equations and define its Traffic Intensity. (3 Marks)

(ii) Consider a drive-in banking service modelled as an M/M/1 queuing system with customer arrival rate of 2 per minute. It is desired to have fewer than 5 customers line up 99% of the time. How fast should the service rate be? (6 Marks)

(iii) Trucks arrive at garage for a stop-over service according to a Poisson process at a rate of one per every 13 minutes, and the garage service time is an exponential rate variable with mean 9 minutes.

(iiia) Find the average number L of trucks, the average time W a truck spends in the garage, and the average time W_q a truck spends in waiting for service. (5 Marks)

(iiib) Due to increased traffic, suppose that the arrival rate of the trucks increases by 5%. Find the corresponding changes in L, W, and W_q. (5 Marks)

(iiic) Discuss your observations. (1 Mark)

END OF EXAMINATION   TOTAL MARKS:240   CONVERT TO 100%